Tracking Non-Stationary Spectral Peak Structure in EEG Data*

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***Abstract*—We develop a particle filter algorithm to simultaneously estimate and track the instantaneous peak frequency, amplitude, and bandwidth of multiple concurrent non-stationary components of an EEG signal in the time-frequency domain. We use this method to characterize human EEG activity during anesthesia-induced unconsciousness.**

I. Introduction

In the analysis of neural EEG data, it is common practice to compute a spectrogram of the data in order to examine the oscillatory behavior of the underlying neural networks during an experimental task or physiological process. The network activity is often observed as a highly structured peak in the time-frequency domain, and may possess non-stationary peak frequency, bandwidth, and amplitude. The time-varying features of these neural rhythms are commonly described qualitatively or quantitatively at several discrete time points. In this paper, we describe a novel quantitative method to simultaneously track instantaneous peak frequency, bandwidth, and amplitude of multiple concurrent non-stationary components of the EEG signal in time-frequency domain. While different methods have been proposed for time-varying characterization of non-stationary signals [1-7], simultaneous time-varying estimates of frequency, amplitude, and bandwidth in multiple spectral modes have, to our knowledge, not been developed previously. We present a particle filter approach to parameter estimation in dynamic models of non-stationary spectral peaks, and apply this method to analyze EEG data during anesthesia-induced unconsciousness.

II. Methods

A. Spectral Peak Parameter State Models

For EEG spectrogram observations over discrete times $t \in \{1,...,T\}$ and fixed-width frequency bins centered at frequencies $f \in \{1,...,F\}$, we can construct a matrix of the spectral domain observations

$$Y = \begin{pmatrix} y_{1,1} & \cdots & y_{T,1} \\ \vdots & \ddots & \vdots \\ y_{1,F} & \cdots & y_{T,F} \end{pmatrix}, \quad (1)$$

such that $y_{t,f}$ is the magnitude of the power spectrum at time t within frequency bin f.

Our goal is to compute an estimate of Y that is constructed from a series of non-stationary spectral peaks with time-varying parameters. To compute $\hat{y}_{t,f}$, the estimate of $y_{t,f}$, we model the spectral observations as a sum of N discrete non-stationary peaks in spectral power, such that

$$\hat{y}_{t,f} = \sum_{n=1}^{N} \Omega_n\left(f, \theta_t^n\right) + \varepsilon_{y_{t,f}}, \quad (2)$$

where $\Omega_n\left(f, \theta_t^n\right)$ is the magnitude of the n^{th} spectral peak, and $\varepsilon_{y_{t,f}}$ is the observation noise at t within frequency bin f. Each spectral peak is characterized at time t by its own set of parameters, θ_t^n, which describes its time-varying amplitude A_t, peak frequency F_t, and bandwidth B_t.

The temporal evolutions of the parameters are defined as random walks, such that for a given parameter x at time t,

$$x_t = x_{t-1} + \varepsilon_{x_t}, \quad (3)$$

where $\varepsilon_{x_t} \sim N\left(0, \sigma_{x_t}^2\right)$.

The state variance parameters are also random walks

$$\sigma_{x_t}^2 = \sigma_{x_{t-1}}^2 + \varepsilon_v, \quad (4)$$

where $\varepsilon_v \sim N\left(0, \sigma_v^2\right)$ and v is a constant.

We also model observation noise as a function of frequency f

$$\varepsilon_{y_{t,f}} = N\left(0, \sigma_{y_t}^2 f^{-1}\right), \quad (5)$$

which reflects the $1/f$ noise phenomenon widely observed in physiological EEG data.

The observation noise variance also evolves as a random walk with constant variance

$$\sigma_{y_t}^2 = \sigma_{y_{t-1}}^2 + \varepsilon_v \quad (6)$$

where $\varepsilon_v \sim N\left(0, \sigma_v^2\right)$ as in (4).

Thus, the parameter vector for each spectral peak is $\theta_t^n = \left\{A_t, F_t, B_t, \sigma_{A_t}^2, \sigma_{F_t}^2, \sigma_{B_t}^2\right\}$.

B. Observation Model Components

Given θ_t^n, we can use the observation model

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$$\Omega_n\left(f,\theta_t^n\right)=\exp\left(A_t-\frac{\left(f-F_t\right)^2}{2B_t}\right) \quad (7)$$

to characterize the spectral peak as a Gaussian with amplitude $\exp\left(A_t\right)$, peak frequency F_t, and bandwidth B_t.

The Gaussian, however, is not always an appropriate model of peak structure. Since frequency, by definition, is bounded at zero, the structure of the spectral peaks with low peak frequencies is highly asymmetrical. This is evident in the EEG slow/delta rhythms seen in sleep and general anesthesia. Therefore we define an alternative model for the peak structure as follows:

$$\Omega_n\left(f,\theta_t^n\right)=\frac{\exp\left(A_t\right)}{\nu}\left[f^{(k-1)}\exp\left(-\frac{f}{\varphi}\right)\right], \quad (8)$$

where

$$\nu=\exp\left(1-k\right)\left(\left(k-1\right)\varphi\right)^{(k-1)}, \quad (9)$$

$$k=\frac{1}{2B_t}\left(\sqrt{F_t^2\left(4B_t+F_t^2\right)}+2B_t+F_t^2\right), \quad (10)$$

and

$$\varphi=\frac{1}{2F_t}\left(\sqrt{F_t^2\left(4B_t+F_t^2\right)}-F_t^2\right). \quad (11)$$

This characterizes the spectral structure based on the shape of a Gamma distribution with amplitude $\exp\left(A_t\right)$, peak frequency F_t, bandwidth B_t, and shape and scale parameters k and φ, respectively.

C. Likelihood and Particle Filter

Given N simultaneous spectral peaks, we can construct a parameter vector at each time t,

$$\Theta_t=\left\{\bigcup_{n=1}^{N}\theta_t^n,\sigma_{y_t}^2\right\}. \quad (12)$$

It should be noted that Θ_t is positive definite.

For a given Θ_t, we can compute $\Pr\left(Y_t\mid\Theta_t\right)$, the probability at time t of the observed data given the model, which is proportional to the instantaneous likelihood,

$$\Pr\left(Y_t\mid\Theta_t\right)\propto L\left(\Theta_t\right)=\exp\left(-\sum_{f=1}^{F}\frac{\left(y_{t,f}-\hat{y}_{t,f}\right)^2}{2\left(\sigma_{y_t}^2 f^{-1}\right)}\right). \quad (13)$$

Using the parameter vector and likelihood, we can build a particle filter. The particle filter is a Bayesian resampling method that generates a set of P parameter vectors (called particles) whose distribution approximates the posterior distribution, $\Pr\left(\Theta_t\mid Y_t\right)$.

The initial particle values are drawn from a pre-defined proposal density. In dealing with EEG data under experimental conditions, we often have the benefit of a strong understanding of the characteristics of the spectral peaks in questions. This allows us to intelligently select priors for each of the parameters in question. For each spectral peak parameter x at time 0, we set

$$\Pr\left(x_0\right)\sim U\left(x_{\min},x_{\max}\right), \quad (14)$$

where each parameter is distributed uniformly between experimentally known bounds. If there is no physiological precedent, a broad uniform may be used. We call the multidimensional proposal density for the entire parameter vector, $\pi\left(\Theta_0\right)$.

The iterative procedure is as follows:

Given a set of P particles, where ρ_t^i is the i^{th} particle at time t, and contains values for Θ_t:

1) Initialize the particles using the proposal densities, such that at $t=0$, $\rho_0^i\sim\pi\left(\Theta_0\right)$
2) For each time $t\in\left\{2,\ldots,T\right\}$, for all particles $\left\{\rho_t^1,\ldots\rho_t^P\right\}$
   a) Sample a new value for each particle based on the one-step prediction density, $\rho_{t|t-1}^i\sim\Pr\left(\Theta_t\mid\rho_{t-1}^i\right)$, defined by (3) and (4). Constrain $\rho_{t|t-1}^i>0$ to ensure that each parameter is positive.
   b) Compute a weight w^i for each particle using an exponentiated (13), such that $w^i=L\left(\rho_{t|t-1}^i\right)$, and normalize so that $\sum_{i=1}^{P}w^i=1$
   c) Resample the collection of particles according to the set of weights $\Pr\left(\rho_t^i=\rho_{t|t-1}^j\right)=w^j$.
   d) We define the parameter estimate $\hat{\Theta}_t$ as the component-wise median of $\left\{\rho_t^1,\ldots\rho_t^P\right\}$. Confidence bounds with at significance α can be computed using the component-wise $\alpha/2$ and $1-\alpha/2$ percentiles of $\left\{\rho_t^1,\ldots\rho_t^P\right\}$.

The estimated filtered spectrogram can be reconstructed using $\hat{\Theta}$ and (2), (7)-(11).

D. Simulated Data

We simulated data with a chirp-like spectrum, a Gaussian bandwidth structure, and $1/f$ observation noise. The simulation was parameterized so that

$$\begin{aligned}F_t&=\exp\left(m_F t+b_F\right)\\B_t&=m_B t+b_B\\A_t&=\exp\left(m_A t+b_A\right)\end{aligned}, \quad (15)$$

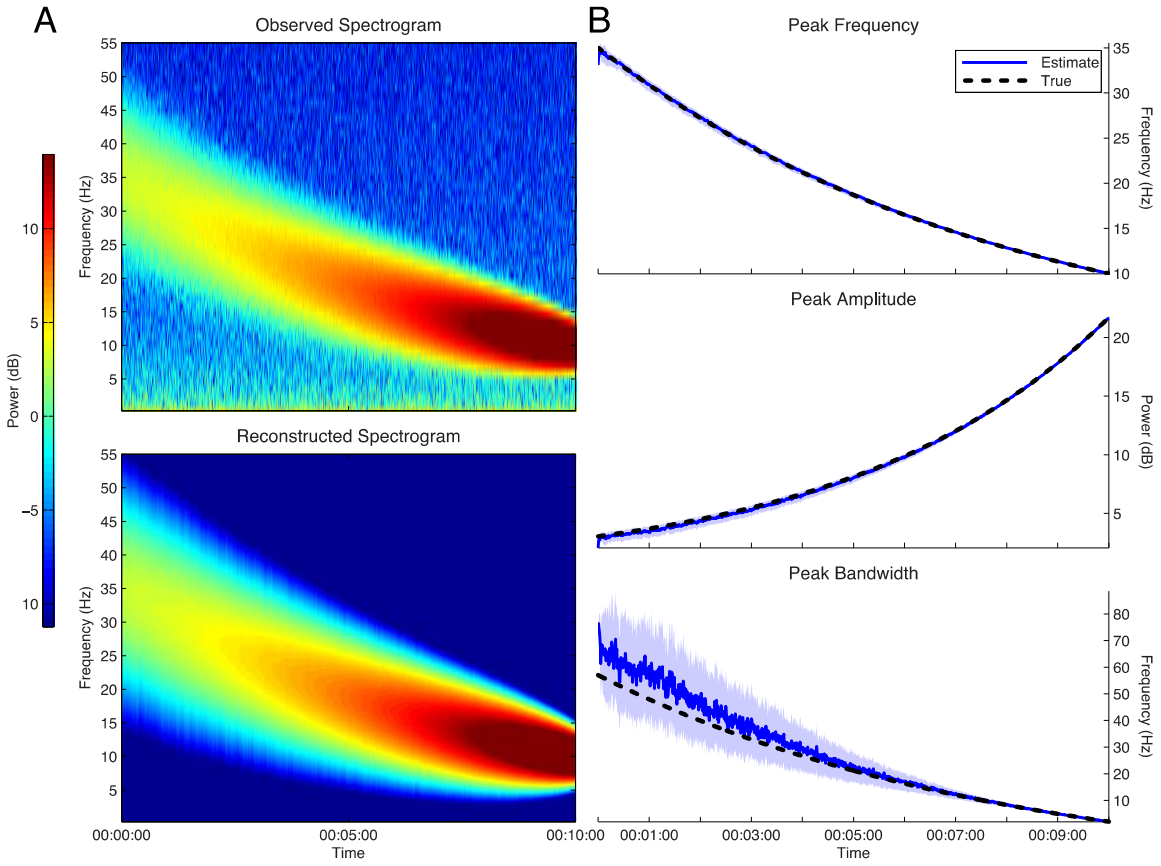

Fig. 1. Simulated data example. The particle filter was run on multitaper spectrogram data generated from a simulated chirp (A, top). In (B), the estimate medians (blue curves), 90th percentiles (light blue regions), and true values (dashed black curves) of the time-varying peak frequency (top), variance (middle), and bandwidth (bottom) parameters are shown, which are used to generate a denoised reconstructed spectrogram (A, bottom).

over 10 minutes time sampled at 2Hz. We set m_F, m_B, and m_A to -1.2528, 2.2, and 1.9661, respectively, and b_F, b_B, and b_A to ln(35), -20, and ln(.7), respectively. Using these parameter values along with (7), noise was added to each frequency using (5) with $\sigma_y^2 = 1$.

E. Experimental Data

For the experimental data example, we used the high-density (64-channel) EEG data set collected in [8], during the administration of general anesthesia under the drug propofol. In this experiment, the subject was brought out and in of consciousness using a computer-controlled infusion pump, which slowly raised the concentration of propofol from a baseline of 0 mcg/ml to a peak level of 5 mcg/ml, then gradually returned the concentration back again to 0 mcg/ml. In our example, we use the EEG data from one subject during an ~2 hour experiment, examining a single laplacian-referenced frontal channel.

III. Results

As a proof of concept, we first applied the particle filter to simulated EEG spectral chirp data with $1/f$ noise (Fig. 1A, top). 10000 particles and broad uniform priors for all parameters were used. The filter output produced time-varying estimates (Fig. 1B, blue curves) of the peak frequency (top panel), amplitude (middle), and bandwidth (bottom). The filter estimates the peak parameters well, with the true value (dashed black curves) falling within the 90% confidence bounds (light blue regions) 99.92%, 99.58%, and 97.58% of the time for the peak frequency, amplitude, and bandwidth, respectively. Consequently, the reconstructed spectrogram (Fig. 1A, bottom) strongly resembles the chirp in the original spectrogram. For the bandwidth, the estimate is initially uncertain, as the signal to noise ratio is low for the large bandwidth/low amplitude portion of the chirp. As the chirp amplitude increases and the bandwidth narrows, the bandwidth confidence bounds become increasingly tighter.

We then applied the particle filter to experimental EEG data during general anesthesia (GA) under propofol (Fig. 2). Propofol data is especially suited for this analysis as it has two major oscillatory modes that change during the administration of GA: the traveling peak and the slow oscillation (Fig. 2A, top). The traveling peak is a rhythm that appears during light anesthesia as a broadband, low amplitude spectral peak at 15-25 Hz, then transitions to a more narrowband, high amplitude spectral peak at 8-12 Hz as the concentration of propofol increases. The slow oscillation is a rhythm centered at <1.5Hz with a highly skewed peak structure, and amplitude that greatly increases during the administration of propofol [8].

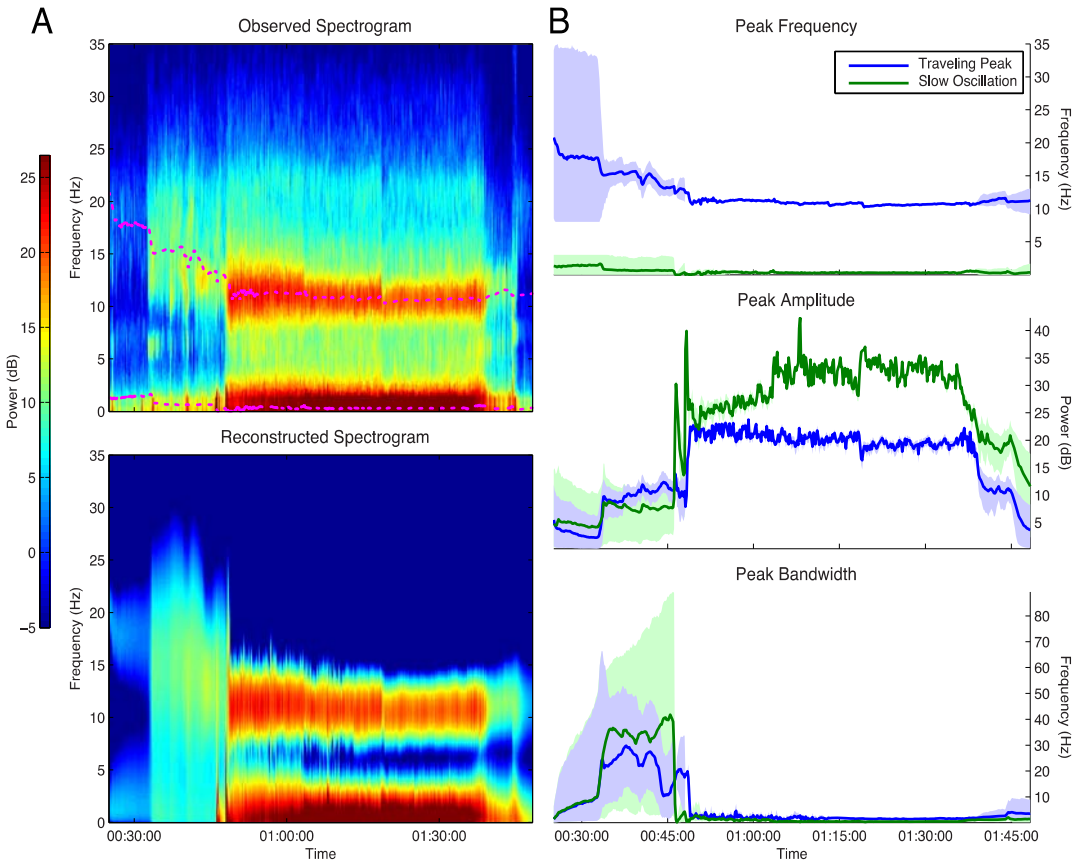

Fig. 2. Experimental EEG data example. The particle filter was run on EEG data during general anesthesia during propofol (A, top). In (B), the estimates of the traveling peak (blue) and slow oscillation (green) medians (solid curves) and 90th percentiles (light regions), of the time-varying peak frequency (top), variance (middle), and bandwidth (bottom) parameters are shown, which are used to generate a denoised reconstructed spectrogram (A, bottom). The peak frequency estimate medians for the slow oscillation and traveling peak are overlaid on the original data (A, top, dashed magenta curves).

The instantiation of the particle filter used 10000 particles to simultaneously estimate both peaks, with Gaussian and gamma structures used for the traveling peak and slow oscillation, respectively. Priors were chosen based on knowledge of the physiological system. For the traveling peak, the peak frequency and bandwidth priors were uniform random 5-35Hz and 0-30Hz, respectively. For the slow oscillation, the peak frequency and bandwidth priors were uniform random 0-5Hz and 0-3Hz, respectively. The amplitude priors were uniform from 0 to the maximum of the log of the data power.

As in the simulation, the particle filter estimates for all three peak parameters (Fig. 2B) had the greatest uncertainty at the beginning and end of the experiment, when the amplitudes were lowest and the bandwidth was large. Overall, the estimates of peak frequency tracked the trends of the data peaks (Fig. 2A, top, magenta dashed curve) for both rhythms. The gamma structure of the slow oscillation model allowed for a reasonable reconstruction of a highly skewed spectral peak (Fig. 2A bottom).

IV. Conclusion

This method provides a powerful tool for the quantitative analysis of EEG data. Studies of relationships between estimated peak parameters and physiological correlates could provide insight into characterizing the dynamic processes governing neural activity.

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